Natural Sciences 102 Problem Set 1 Solutions

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Problem 3

Part a

Well, I think that to give a sensible answer to this question we should first clarify how unaided is the "unaided eye", because I am so shortsighted that if my eyes were completely unaided then I wouldn't even be able to count how many toes I have. So let's agree that the unaided eye assumes perfect 20/20 vision. Then I would estimate that in a clear night it is probably possible to see hundreds of stars (just think about all the constellations you can see).

Part b

We could solve this problem right away if we knew the mass of the Sun and the mass of an hydrogen atom, because assuming that the Sun is made *only* of hydrogen then the ratio of the former by the latter would yield immediately how many atoms there are in the Sun. For the sake of the argument, let's suppose we're in the XIX century, we don't have google to tell us what is the measured mass of the Sun and we want to come up with an estimate for that... after all putting it on a scale is not such a practical approach. On the other hand, measuring the mass of an hydrogen atom has never been such a complicated task, so we can assume that we know it to be $m_H = 1.67 \cdot 10^{-27}$ kg.

Let's then estimate the mass of the Sun. That is not such a complicated task. We know the Sun's angular size θ and we can measure the distance from the Earth to the Sun D_{ES} (remember the cosmic distance ladder mentioned in class). The radius of the Sun is then just $R_S = D_{ES} \tan(\theta/2) \approx D_{ES} \cdot \frac{\theta}{2}$. Knowing the radius of the Sun and knowing that it is spherical we can estimate its volume V_S

$$V_S = \frac{4\pi}{3} R_S^3. \tag{1}$$

Now how do we convert a volume into a mass? Well, the *density* (usually denoted by the greek letter ρ) of an object is the ratio between its mass and volume. So if we multiply the volume by the density we get the mass. Now what is the density of the Sun? Remember that we're just doing an order of magnitude estimate, therefore we don't have to be very precise, so let's assume that the density of the Sun is equal to the density of water, that is $\rho_{H_2O} = 10^3$ kg$/m^3$. Therefore

$$M_S = \frac{4\pi}{3} (D_{ES}\theta)^3 \rho_{H_2O}. \tag{2}$$

So, assuming that $\theta = 0.5^o$ and $D_{ES} = 1.5 \cdot 10^{11}$ m., we have $R_S = 6.5 \cdot 10^8$ m. (not bad compared to the true value $R_S = 6.9 \cdot 10^8$ m.). Then the mass of the Sun is given by $M_S = 1.7 \cdot 10^{30}$ kg. (true value is $M_S = 1.9 \cdot 10^{30}$). Finally, we have that the number of hydrogen atoms in the Sun is then given by

$$N_H = \frac{M_S}{m_H} = \frac{1.7 \cdot 10^{30}}{1.67 \cdot 10^{-27}} = 7.3 \cdot 10^{56}. \tag{3}$$

Part c

Let's suppose that the life of an average person is 70 years. We know that there are 365 days in every year, 24 hours in every day, 60 minutes in every hour and 60 seconds in every minute. Then

$$\begin{aligned} \text{70 years} \quad &\times \quad \frac{\text{365 days}}{\text{1 year}} \times \frac{\text{24 hours}}{\text{1 day}} \times \frac{\text{60 minutes}}{\text{1 hour}} \times \frac{\text{60 seconds}}{\text{1 minute}} \\ &= \quad 2.208 \times 10^9 \text{ sec.} \end{aligned} \tag{4}$$

Two billions seconds! Enjoy every one of them!